DIFFERENTIATION OF COMBINATION OF FUNCTIONS

At Merit level you can be asked to differentiate functions which require the use of a combination of the product rule, the quotient rule and the chain rule on trig, exponential and logarithmic functions.

Examples:

1) Differentiate $y = (2x^2 - 5)^4(3x + 7)$

$f = (2x^2 - 5)^4$, $g = (3x + 7)$

Apply the chain rule on $f = (2x^2 - 5)^4$: $\quad f' = 4(2x^2 - 5)^3(4x)$

Apply the product rule: $y = f' \cdot g + g' \cdot f$

$$= 4(2x^2 - 5)^3(4x) \cdot (3x + 7) + 3 \cdot (2x^2 - 5)^4$$

2) Differentiate $y = 5x^3 \cos(\sqrt{x})$

$f = 5x^3$, $g = \cos(\sqrt{x})$

Apply the chain rule on the composite function $g = \cos(\sqrt{x})$: $\quad g' = {}^-\sin(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}$

Apply the product rule: $y = f' \cdot g + g' \cdot f$

$$= 15x^2 \cdot \cos(\sqrt{x}) + {}^-\sin(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2} \cdot 5x^3$$

3) Differentiate $y = \frac{\sqrt[3]{x}}{\sin(x)}$

$f = \sqrt[3]{x}$, $g = \sin(x)$

Apply the quotient rule: $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$

$$= \frac{\frac{1}{3}x^{-2/3} \cdot \sin(x) - \cos(x) \cdot \sqrt[3]{x}}{\sin^2(x)}$$

Delta Ex 9.6 pg 99, Extension Exercise 9 pg 99